SOLUTION

a)

$$x = 3 \quad or \quad x = -\tfrac{2}{5}$$
$$x - 3 = 0 \quad or \quad x + \tfrac{2}{5} = 0 \qquad \text{Getting 0's on one side}$$
$$(x - 3)\left(x + \tfrac{2}{5}\right) = 0 \qquad \text{Using the principle of zero products (multiplying)}$$
$$x^2 + \tfrac{2}{5}x - 3x - 3 \cdot \tfrac{2}{5} = 0 \qquad \text{Multiplying}$$
$$x^2 - \tfrac{13}{5}x - \tfrac{6}{5} = 0 \qquad \text{Combining like terms}$$
$$5x^2 - 13x - 6 = 0 \qquad \text{Multiplying both sides by 5 to clear fractions}$$

Note that multiplying both sides by the LCD, 5, clears the equations of fractions. Had we preferred, we could have multiplied $x + \frac{2}{5} = 0$ by 5, thus clearing fractions *before* using the principle of zero products.

b)

$$x = 2i \quad or \quad x = -2i$$
$$x - 2i = 0 \quad or \quad x + 2i = 0 \qquad \text{Getting 0's on one side}$$
$$(x - 2i)(x + 2i) = 0 \qquad \text{Using the principle of zero products (multiplying)}$$
$$x^2 - (2i)^2 = 0 \qquad \text{Finding the product of a sum and a difference}$$
$$x^2 - 4i^2 = 0$$
$$x^2 + 4 = 0 \qquad i^2 = -1$$

c)

$$x = 5\sqrt{7} \quad or \quad x = -5\sqrt{7}$$
$$x - 5\sqrt{7} = 0 \quad or \quad x + 5\sqrt{7} = 0 \qquad \text{Getting 0's on one side}$$
$$\left(x - 5\sqrt{7}\right)\left(x + 5\sqrt{7}\right) = 0 \qquad \text{Using the principle of zero products}$$
$$x^2 - \left(5\sqrt{7}\right)^2 = 0 \qquad \text{Finding the product of a sum and a difference}$$
$$x^2 - 25 \cdot 7 = 0$$
$$x^2 - 175 = 0$$

d)

$$x = -4 \quad or \quad x = 0 \quad or \quad x = 1$$
$$x + 4 = 0 \quad or \quad x = 0 \quad or \quad x - 1 = 0 \qquad \text{Getting 0's on one side}$$
$$(x + 4)x(x - 1) = 0 \qquad \text{Using the principle of zero products}$$
$$x(x^2 + 3x - 4) = 0 \qquad \text{Multiplying}$$
$$x^3 + 3x^2 - 4x = 0$$

Try Exercise 29.

Teaching Tip

Students tend to ignore the factor x. In Example 2(d), you may want to emphasize that if 0 is a solution, then x is a factor.

8.3 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *Complete each of the following statements.*

1. In the quadratic formula, the expression $b^2 - 4ac$ is called the __discriminant__.

2. When $b^2 - 4ac$ is 0, there is/are __one__ solution(s).

3. When $b^2 - 4ac$ is positive, there is/are __two__ solution(s).

4. When $b^2 - 4ac$ is negative, there is/are __two__ solution(s).

5. When $b^2 - 4ac$ is a perfect square, the solutions are __rational__ numbers.

6. When $b^2 - 4ac$ is negative, the solutions are __imaginary__ numbers.

For each equation, determine what type of number the solutions are and how many solutions exist.

7. $x^2 - 7x + 5 = 0$ Two irrational

8. $x^2 - 5x + 3 = 0$ Two irrational

9. $x^2 + 3 = 0$ Two imaginary

10. $x^2 + 5 = 0$ Two imaginary

11. $x^2 - 5 = 0$ Two irrational

12. $x^2 - 3 = 0$ Two irrational

13. $4x^2 + 8x - 5 = 0$ Two rational

14. $4x^2 - 12x + 9 = 0$ One rational

15. $x^2 + 4x + 6 = 0$ Two imaginary

16. $x^2 - 2x + 4 = 0$ Two imaginary

17. $9t^2 - 48t + 64 = 0$ One rational

18. $6t^2 - 19t - 20 = 0$ Two rational

Aha! **19.** $9t^2 - 3t = 0$ Two rational

20. $4m^2 + 7m = 0$ Two rational

21. $x^2 + 4x = 8$ Two irrational

22. $x^2 + 5x = 9$ Two irrational

23. $2a^2 - 3a = -5$ Two imaginary

24. $3a^2 + 5 = 7a$ Two imaginary

25. $7x^2 = 19x$ Two rational

26. $5x^2 = 48x$ Two rational

27. $y^2 + \frac{9}{4} = 4y$ Two irrational

28. $x^2 = \frac{1}{2}x - \frac{3}{5}$ Two imaginary

Write a quadratic equation having the given numbers as solutions.

29. $-7, 3$ $x^2 + 4x - 21 = 0$

30. $-6, 4$ $x^2 + 2x - 24 = 0$

31. 3, only solution (*Hint*: It must be a repeated solution.) $x^2 - 6x + 9 = 0$

32. -5, only solution $x^2 + 10x + 25 = 0$

33. $-1, -3$ $x^2 + 4x + 3 = 0$

34. $-2, -5$ $x^2 + 7x + 10 = 0$

35. $5, \frac{3}{4}$ $4x^2 - 23x + 15 = 0$

36. $4, \frac{2}{3}$ $3x^2 - 14x + 8 = 0$

37. $-\frac{1}{4}, -\frac{1}{2}$ $8x^2 + 6x + 1 = 0$

38. $\frac{1}{2}, \frac{1}{3}$ $6x^2 - 5x + 1 = 0$

39. $2.4, -0.4$ $x^2 - 2x - 0.96 = 0$

40. $-0.6, 1.4$ $x^2 - 0.8x - 0.84 = 0$

41. $-\sqrt{3}, \sqrt{3}$ $x^2 - 3 = 0$

42. $-\sqrt{7}, \sqrt{7}$ $x^2 - 7 = 0$

43. $2\sqrt{5}, -2\sqrt{5}$ $x^2 - 20 = 0$

44. $3\sqrt{2}, -3\sqrt{2}$ $x^2 - 18 = 0$

45. $4i, -4i$ $x^2 + 16 = 0$

46. $3i, -3i$ $x^2 + 9 = 0$

47. $2 - 7i, 2 + 7i$ $x^2 - 4x + 53 = 0$

48. $5 - 2i, 5 + 2i$ $x^2 - 10x + 29 = 0$

49. $3 - \sqrt{14}, 3 + \sqrt{14}$ $x^2 - 6x - 5 = 0$

50. $2 - \sqrt{10}, 2 + \sqrt{10}$ $x^2 - 4x - 6 = 0$

51. $1 - \frac{\sqrt{21}}{3}, 1 + \frac{\sqrt{21}}{3}$ $3x^2 - 6x - 4 = 0$

52. $\frac{5}{4} - \frac{\sqrt{33}}{4}, \frac{5}{4} + \frac{\sqrt{33}}{4}$ $2x^2 - 5x - 1 = 0$

Write a third-degree equation having the given numbers as solutions.

53. $-2, 1, 5$ ⊡

54. $-5, 0, 2$ ⊡

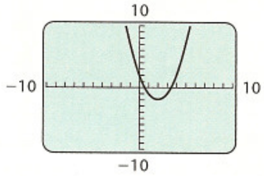
55. $-1, 0, 3$ ⊡

56. $-2, 2, 3$ ⊡

TW **57.** Explain why there are not two different solutions when the discriminant is 0.

⊡ Answers to Exercises 53–56 are on p. IA-17.

TW **58.** While solving a quadratic equation of the form $ax^2 + bx + c = 0$ with a graphing calculator, Amberley gets the following screen. How could the sign of the discriminant help her check the graph?



SKILL REVIEW

To prepare for Section 8.4, review solving formulas and solving motion problems (Sections 3.3, 6.5, and 6.8).

Solve each formula for the specified variable. [6.8]

59. $\frac{c}{d} = c + d$, for c $c = \frac{d^2}{1 - d}$

60. $\frac{p}{q} = \frac{a + b}{b}$, for b $b = \frac{aq}{p - q}$

61. $x = \frac{3}{1 - y}$, for y $y = \frac{x - 3}{x}$, or $1 - \frac{3}{x}$

Solve.

62. *Boating.* Kiara's motorboat took 4 hr to make a trip downstream with a 2-mph current. The return trip against the same current took 6 hr. Find the speed of the boat in still water. [3.3] 10 mph

63. *Walking.* Jamal walks 1.5 mph faster than Kade. In the time it takes Jamal to walk 7 mi, Kade walks 4 mi. Find the speed of each person. [6.5] Jamal: 3.5 mph; Kade: 2 mph

64. *Aviation.* Taryn's Cessna travels 120 mph in still air. She flies 140 mi into the wind and 140 mi with the wind in a total of 2.4 hr. Find the wind speed. [6.5] 20 mph

SYNTHESIS

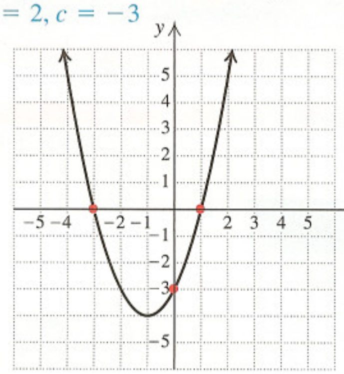
TW 65. If we assume that a quadratic equation has integers for coefficients, will the product of the solutions always be a real number? Why or why not?

TW 66. Can a fourth-degree equation have exactly three irrational solutions? Why or why not?

67. The graph of an equation of the form

$$y = ax^2 + bx + c$$

is a curve similar to the one shown below. Determine a, b, and c from the information given. $a = 1, b = 2, c = -3$



68. Show that the product of the solutions of $ax^2 + bx + c = 0$ is c/a. ⊡

For each equation under the given condition, **(a)** *find k and* **(b)** *find the other solution.*

69. $kx^2 - 2x + k = 0$; one solution is -3 **(a)** $-\frac{3}{5}$; **(b)** $-\frac{1}{3}$

70. $x^2 - kx + 2 = 0$; one solution is $1 + i$ **(a)** 2; **(b)** $1 - i$

71. $x^2 - (6 + 3i)x + k = 0$; one solution is 3 **(a)** $9 + 9i$; **(b)** $3 + 3i$

⊡ Answers to Exercises 68, 72, and 73 are on p. IA-17.

72. Show that the sum of the solutions of $ax^2 + bx + c = 0$ is $-b/a$. ⊡

73. Show that whenever there is just one solution of $ax^2 + bx + c = 0$, that solution is of the form $-b/(2a)$. ⊡

74. Find h and k, where $3x^2 - hx + 4k = 0$, the sum of the solutions is -12, and the product of the solutions is 20. (*Hint*: See Exercises 68 and 72.) $h = -36, k = 15$

75. Suppose that $f(x) = ax^2 + bx + c$, with $f(-3) = 0$, $f\left(\frac{1}{2}\right) = 0$, and $f(0) = -12$. Find a, b, and c. $a = 8, b = 20, c = -12$

76. Find an equation for which $2 - \sqrt{3}$, $2 + \sqrt{3}$, $5 - 2i$, and $5 + 2i$ are solutions. $x^4 - 14x^3 + 70x^2 - 126x + 29 = 0$

Aha! 77. Write a quadratic equation with integer coefficients for which $-\sqrt{2}$ is one solution. $x^2 - 2 = 0$

78. Write a quadratic equation with integer coefficients for which $10i$ is one solution. $x^2 + 100 = 0$

79. Find an equation with integer coefficients for which $1 - \sqrt{5}$ and $3 + 2i$ are two of the solutions. $x^4 - 8x^3 + 21x^2 - 2x - 52 = 0$

TW 80. A discriminant that is a perfect square indicates that factoring can be used to solve the quadratic equation. Why?

Try Exercise Answers: Section 8.3

7. Two irrational **29.** $x^2 + 4x - 21 = 0$

8.4 Applications Involving Quadratic Equations

- Solving Problems
- Solving Formulas

SOLVING PROBLEMS

Some problems translate to rational equations. The solution of such rational equations can involve quadratic equations.

EXAMPLE 1 **Motorcycle Travel.** Keisha rode her motorcycle 300 mi at a certain average speed. Had she averaged 10 mph more, the trip would have taken 1 hr less. Find Keisha's average speed.